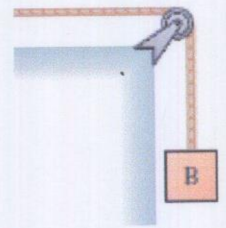
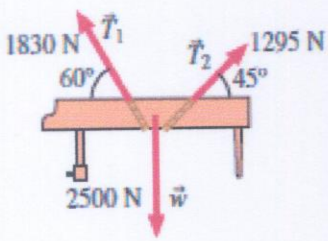
Group Problems: work and energy

1. You raise box B (m = 1.2 kg) from the floor to a height of 2.0 m by supplying a constant tension of 15 N from your end of the rope. Find the speed of the box as it reaches the 2.0 m height. Do you expect the velocity to be constant as it rises? Check your work by solving again using Newton's laws and kinematics.



2. The two ropes shown are being used to lower a 255 kg piano exactly 5 m from a second-story window to the ground. How much work is done by each of the forces? Is the piano speeding up, slowing down, or moving at a constant speed as it descends?

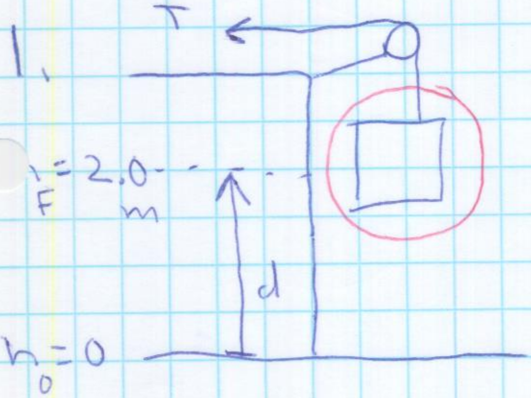


3. A cyclist is coasting at 12 m/s when she starts down a 450 m long slope that is 30 m high. The cyclist and her bicycle have a combined mass of 70 kg. A steady 12 N drag force due to air resistance acts on her as she coasts all the way to the bottom. What is her speed at the bottom?

4. A 1000 kg car is traveling along a straight horizontal road with a speed of 24 m/s when the driver suddenly hits the brakes to avoid a deer in the road ahead. The coefficient of kinetic friction between tires and road is 0.37. What is the car's stopping distance?

5. A 1000 kg car is driving up a hill which is angled 30^0 up from the horizontal. The coefficient of kinetic friction is 0.37. If the car has a speed of 24 m/s when the driver hits the brakes, what is the stopping distance? Give a physical reason why the stopping distance is less than it was on level ground.

1.



System: Box + center of Earth.

Work: done by Tension

$$K_F + U_{gF} + \Delta E_{TH} = K_0 + U_{g0} + W$$

$$\frac{1}{2} m v_F^2 + mgh_F + 0 = 0 + 0 + Td\cos\theta$$

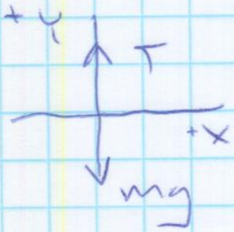
(At rest on floor.)

$$\frac{1}{2}(1.2\text{ kg})v_F^2 + (1.2\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m}) = (15\text{ N})(2\text{ m})\cos 0^\circ$$

$$0.6\, v_F^2 + 23.52 = 30$$

$$v_F = 3.3\text{ m/s}$$

Work done by T adds energy to system. $W = \Delta KE$, so velocity is increasing.



$$\Sigma F = ma$$

$$T - w = ma$$

$$15 - (1.2)(9.8) = 1.2a$$

$$a = +2.7\text{ m/s}^2$$

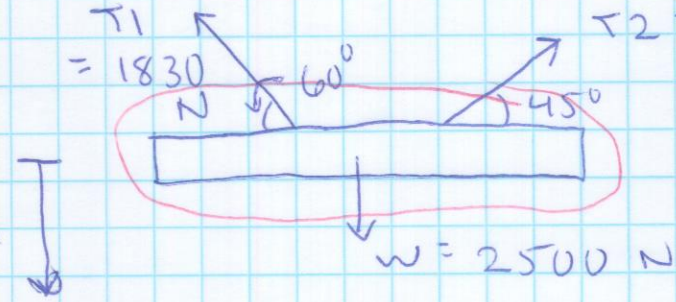
(+), so accel. is up

$$v_F^2 = v_0^2 + 2a\Delta x$$

$$v_F^2 = 0 + 2(2.7\text{ m/s}^2)(2.0\text{ m})$$

$$v_F = 3.3\text{ m/s} \quad ✓$$

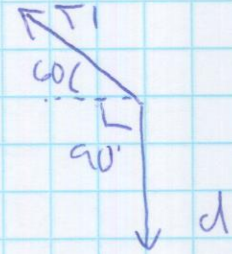
2.



System: Piano.

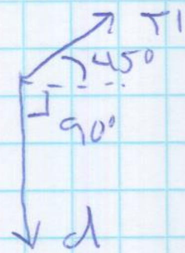
Work: done by T1, T2, w.

$W_F = F d \cos\theta$



T1: $W = (1830\ N)(5\ m)\cos 150°$
$W = -7920\ J$

T2: $W = (1295\ N)(5\ m)\cos 135°$
$W = -4580\ J$



Weight:
$W = (2500\ N)(5\ m)\cos 0°$
$W = +12,500\ J$

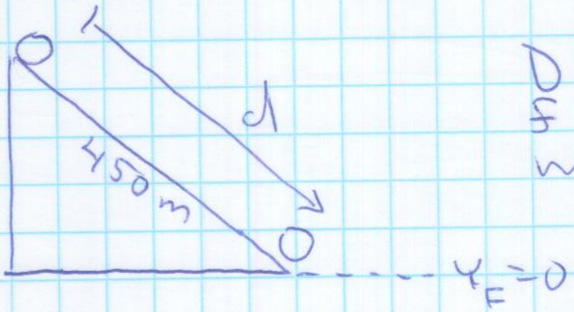
total Work = $W_{T1} + W_{T2} + W_w$

and $W = \Delta K$

$-7920 + -4580 + 12,500 = \Delta K$

$\Delta K = 0$.

So v = constant



$D = 12\ N$

$f = 0$

$m = 70\ kg$

$$K_F + U_{gF} + \Delta E_{TH} = K_0 + U_{g0} + W$$

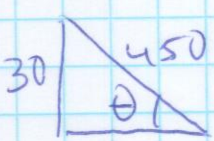
$$\frac{1}{2}mv_F^2 + 0 + D\Delta x = \frac{1}{2}mv_0^2 + mgy_0 + \varnothing$$

$$v_F^2 + 2\frac{D\Delta x}{m} = v_0^2 + 2gy_0$$

$$v_F^2 + 2(12)(450)/70 = (12)^2 + 2(9.8)(30)$$

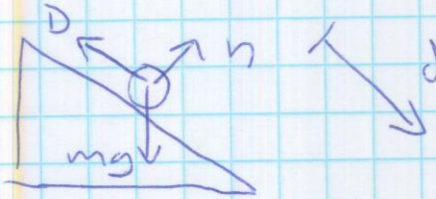
$$v_F = 24\ m/s$$

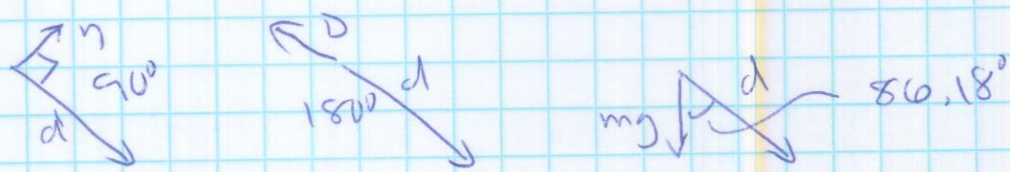
II: Only cyclist / bike in system. All forces do work.



$\sin\theta = 30/450$

$\theta = 3.82°$





$W_n = 0$

$$W_D = D\,d\cos\theta$$
$$= (12)(450)\cos 180 = -5400\ J$$

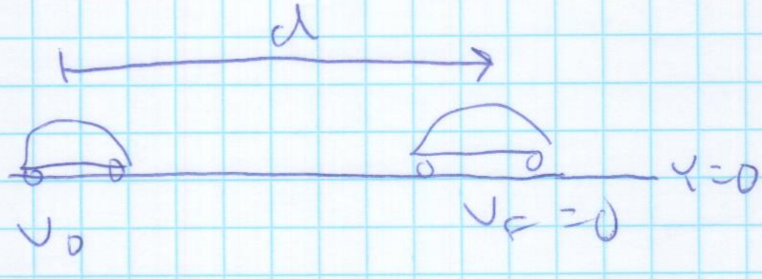
$$W_{mg} = (mg)\,d\cos\theta$$
$$= (70)(9.8)(450)\cos 86.18 = 20566\ J$$

$$\text{total } W = 20566 - 5400 + 0 = 15166\ J$$

$$W = \Delta KE$$
$$15166 = \frac{1}{2}(70)v_F^2 - \frac{1}{2}(70)(12)^2$$

$$v_F = 24\ m/s$$

4.

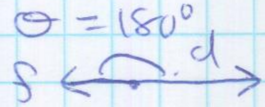


System: Cart
Center of Earth

Work: done by f_k.

$$K_F + U_{gF} + \Delta E_{TH} = K_0 + U_{g0} + W$$

$$0 + 0 + 0 = \frac{1}{2}mv_0^2 + 0 + f\,d\cos\theta$$



$$\frac{1}{2}mv_0^2 + (\mu n)\,d\cos 180° = 0$$

$$\Sigma F_y = ma_y$$
$$n - mg = 0$$
$$n = mg$$

$$\frac{1}{2}mv_0^2 + \mu mg\,d(-1) = 0$$

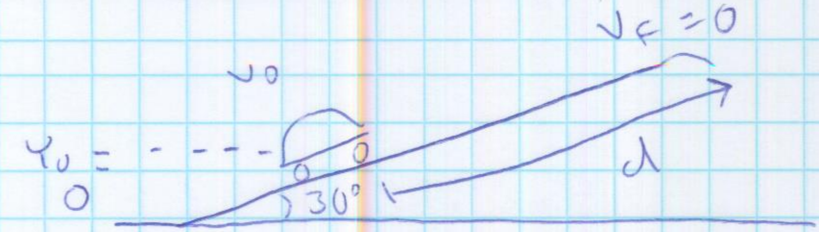
$$\frac{1}{2}v_0^2 = \mu g d$$

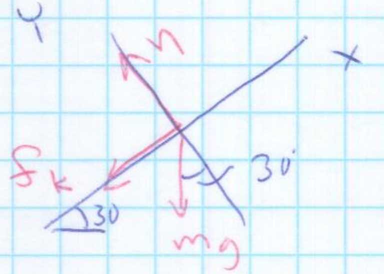
$$\frac{1}{2}(24)^2 = 0.37(9.8)\,d$$

$$d = 79 \text{ m}$$

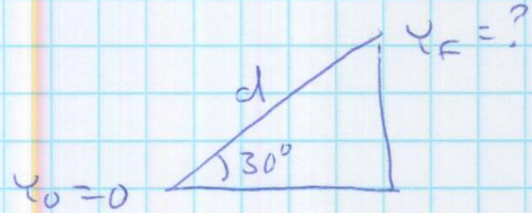
[illegible] system is car + Earth.

Friction does work.





PREP:



$$\sin 30° = \frac{y_F}{d}$$

$$y_F = d \sin 30°$$

$$\Sigma F_y = m a_y$$

$$n - mg\cos 30° = 0$$

$$n = mg\cos 30°$$

$$K_F + U_{gF} + \Delta E_{TH} = K_0 + U_{g0} + W$$

$$0 + mgy_F + 0 = \tfrac{1}{2}mv_0^2 + 0 + f\,d\cos 180°$$

$$mg(d\sin 30°) = \tfrac{1}{2}mv_0^2 + \mu n\, d\,(-1)$$

$$mg(d\sin 30°) = \tfrac{1}{2}mv_0^2 + {}^{-}\mu(mg\cos 30°)\,d$$

$$d[mg\sin 30 + \mu mg\cos 30] = \tfrac{1}{2}mv_0^2$$

$$d[g\sin 30 + \mu g\cos 30] = \tfrac{1}{2}mv_0^2$$

$$d[9.8\sin 30 + 0.37(9.8)\cos 30] = \tfrac{1}{2}(24)^2$$

$$d = 36\text{ m}$$

Less than on flat because some K_0 goes into increased U_g.

Or, because weight has a component which tends to pull car backward.