Heat Transfer Problems

general advice: decide whether the heat transfer is occurring by convection, conduction, or radiation. It is possible that a problem may involve several.

If radiation is involved, are you interested in the power radiated, absorbed, or the net?

1. How many days does it take for a perfect blackbody cube 0.010 m on a side at 30 ^{0}C to radiate the same amount of energy that a 100 W lightbulb uses in one hour?

2. A car parked in the sun absorbs energy at a rate of 500 J/s $m^{2.}$ The car will eventually reach a temperature at which it radiates energy at this same rate. If the car has an emissivity of 0.90, what is this temperature?

3. A person's body is producing energy internally due to metabolic processes. If the body loses more energy than metabolic processes are producing, its temperature will drop. If the drop is severe, it can be life-threatening. Suppose that a person is unclothed and energy is being lost via radiation from a body surface area of 1.40 m^2, which has a temperature of 34 ^{0}C and an emissivity of 0.70. Also suppose that metabolic processes are producing energy at a rate of 115 J/s. What is the temperature of the coldest room in which this person could stand and not experience a drop in body temperature?

4. In a house the temperature at the surface of a window is 25 ^{0}C. The temperature outside the window is 5 ^{0}C. Heat is lost through the window by conduction, and the heat lost per second has a certain value. The temperature outside begins to fall, while the conditions inside the house remain the same. As a result, the rate of heat loss increases. What is the temperature outside the window when the heat lost per second has doubled?

5. An old house uses hot water radiators. In one room, the radiator has a dark color (e = 0.75). The temperature of the radiator is 62 ^{0}C. A new owner paints the radiator a lighter color (e = 0.50). If the radiator is to emit the same radiant power as it did before it was painted, what will its temperature need to be now?

6. One end of a brass bar (k = 110 W/m K) is maintained at 306 ^{0}C, while the other end is kept at a constant, but lower, temperature. The cross-sectional area of the bar is 2.6 x 10^{-4} m^2. The sides of the bar are insulated, so no heat is lost through the sides. Heat flows through the bar at a rate of 3.6 J/s. What is the temperature of the bar 0.15 m from the hot end?

1. Want to know about its Q emitted only.

$$Q_{emitted} = e\sigma A T^4 \Delta t$$

want $Q_{em} = (100\ J/s)(3600 s)$
$= 3.6 \times 10^5\ J$

Surface area of cube = 6 × Area of one side

$$A = 6(0.01 \times 0.01)\ m^2$$
$$= 6 \times 10^{-4}\ m^2$$

$$\Delta t = \frac{Q}{e\sigma A T^4} = \frac{3.6 \times 10^5\ J}{(1)(5.67 \times 10^{-8})(6 \times 10^{-4})(303)^4}$$

$\Delta t = 1.26 \times 10^6$ s

$= 350$ hours

$= 14.6$ days

2. $\frac{Q}{A \Delta t} = e \sigma T^4_{ENV} = 500 \text{ J/s m}^2$

so eventually $\frac{Q}{A \Delta t}\Big]_{EMITTED} = 500 \text{ W/m}^2$

– now car is at radiative equilibrium : net $Q = 0$

$e \sigma T^4_{CAR} = 500 \text{ J/s m}^2$

$(0.90)(5.67 \times 10^{-8}) T^4_{CAR} = 500$

$T_{CAR} = 315 \text{ K}$

$= 42^\circ \text{ C}$

7. Metabolism adds $Q/t = 115$ J/s.

Person absorbs $Q/t = e\sigma A T_0^4$ from room at T_0.

Person emits $Q/t = e\sigma A T^4$ due to own body temp.: $T = 34°C$

$T = 307$ K

Emits $Q/t = (0.70)(5.67 \times 10^{-8})(1.40)(307)^4$

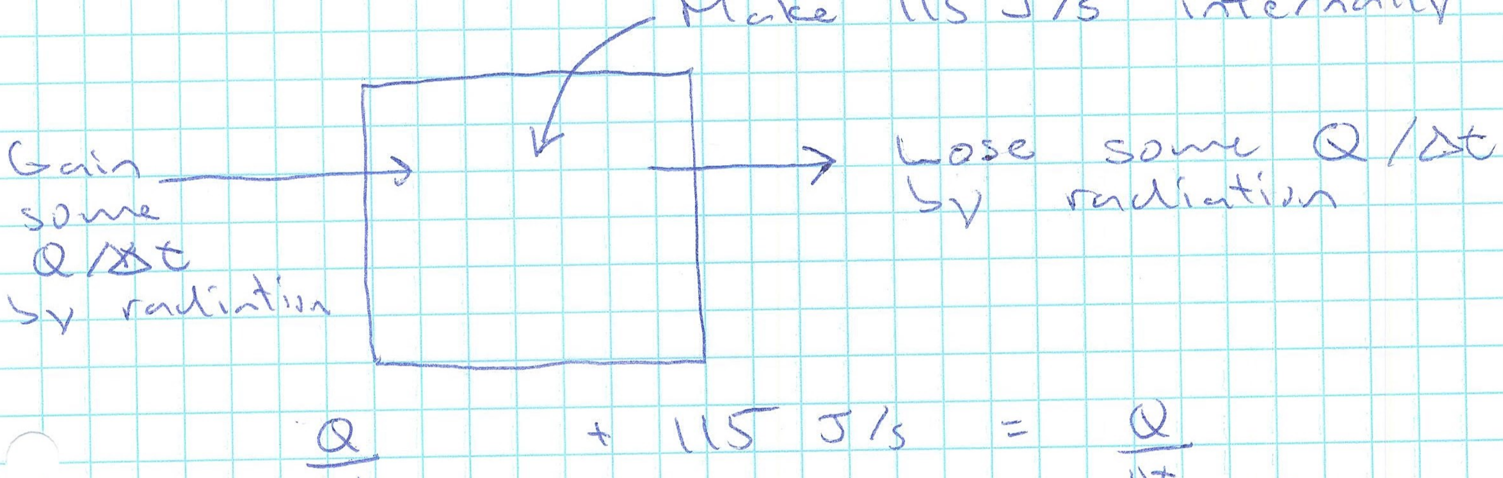
Absorbs $Q/t = (0.70)(5.67 \times 10^{-8})(1.40)\, T_0^4$

Can emit 115 J/s more than absorbing and neither have net gain or loss

$$Q/t\,]_{EMITTED} - Q/t\,]_{Abs} = 115 \text{ J/s}$$

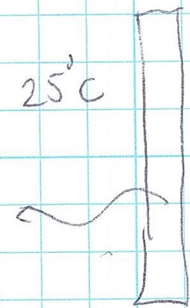
$$(0.70)(5.67 \times 10^{-8})(1.40)\left[\, 307^4 - T_0^4 \,\right] = 115$$

$$T_0 = 287 \text{ K}$$
$$= 14° \text{ C}$$



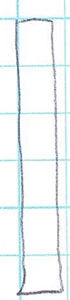
$$\frac{Q}{\Delta t}_{GAINED} + 115 \text{ J/s} = \frac{Q}{\Delta t}_{LOST}$$

4

$T_{IN} = 25°C$  $T_{OUT} = 5°C$

Window has some A, k, L

old $\frac{Q}{t} = \frac{kA(25-5)}{L}$

new $\frac{Q}{t} = \frac{kA(25 - T_{OUT})}{L}$ $T_{IN} = 25°C$  $T_{OUT} = ?$

and $\left.\frac{Q}{t}\right)_{new} = 2 \left.\frac{Q}{t}\right)_{OLD}$

$\frac{kA(25-T)}{L} = 2\,\frac{kA(25-5)}{L}$

$25 - T = 40$

$T = -15°\ C$

5. ~~bo~~ ed

~~2~~. Before, when dark color:

$Q/t = e\sigma A T^4$ is power emitted

$Q/tA = (0.75)(5.67 \times 10^{-8})(62+273)^4$

↑ we don't know area, but it does not change!

$Q/tA = 536 \; J/s\,m^2$

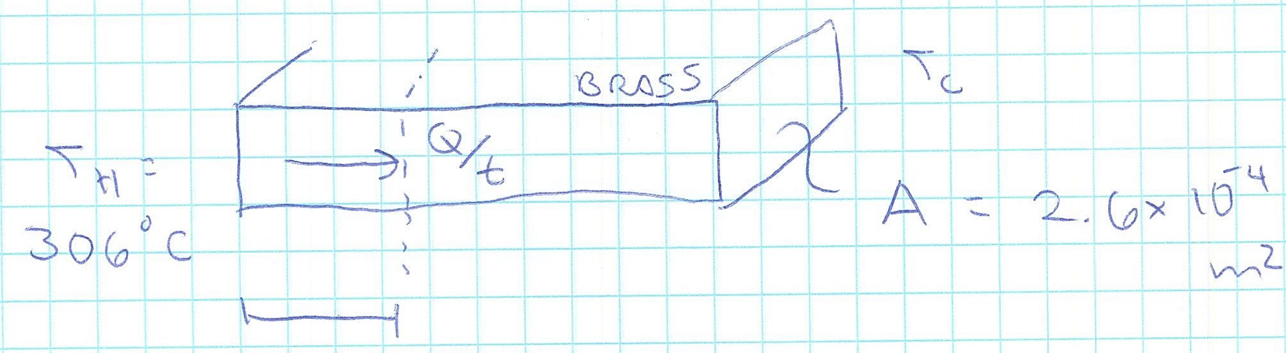
Light color, new $e = 0.50$

$Q/tA = e\sigma T^4$

$536 = (0.5)(5.67 \times 10^{-8}) T^4$

$T = 370 \; K$
$= 98^0 C$ ↖ almost boiling!

6.



$x = 0.15$ m

$T_x = ?$

$Q/\Delta t = 3.6$ J/s

$$Q/\Delta t = \frac{kA\Delta T}{L} = 3.6 \text{ J/s}$$

$$\frac{(110 \text{ J/s m C°})(2.6 \times 10^{-4} \text{ m}^2)(306 - T_x)}{0.15 \text{ m}}$$

$T_x = 287°$ C